

# Fundamentals of Algorithms

## CS502-Fall2011

### ASSIGNMENT1

#### Deadline

Your assignment must be uploaded/submitted at or before **1<sup>st</sup> November 2011**.

#### Uploading instructions

Please view the **assignment submission process** document provided to you by the Virtual University to upload the assignment.

#### Rules for Marking

It should be clear that your assignment will not get any credit if:

- The assignment is submitted after due date.
- The submitted assignment does not compile or run.
- The assignment is copied.**

#### Objectives

This assignment will help you to understand the concepts of Asymptotic Growth, making analysis of pseudo code, recurrence relation development, asymptotic function understanding and iterative solutions for recurrences.

#### Guidelines

#### **RULES FOR CALCULATING TIME COMPLEXITY AND BIG-OH**

##### **Rule 00**

Normally these formulas are very handy:

If  $x^y = z$  then  $y = \log_x z$

Also,

$$\sum_{i=1}^n a_i = \frac{n}{2}(a_1 + a_n) \qquad \sum_{i=1}^n i = \frac{n}{2}(n+1) \qquad \sum_{k=0}^m r^k = \frac{1-r^{m+1}}{1-r}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \text{ (for } n \geq 1)$$

### Rule 0

The condition that stops a loop executes ONE MORE time than the loop itself (the last time is when it is evaluated false)

### Rule 1

for (i=0; i<n; i=i+k) Anything inside the loop will run approximately **n/k** times

### Rule 2

for (i=n; i>0; i=i-k) Anything inside the loop will run approximately **n/k** times

### Rule 3

for (i=1; i<n; i=i\*k) Anything inside the loop will run approximately **log<sub>k</sub>n** times

### Rule 4

for (i=1; i<=n; ++i)  
    for (j=1; j<=i; ++j)

The above nested loop approximately runs  $\frac{1}{2} n(n+1)$  times.

The variable j depends upon the value of i

### Rule 5

for (i=1; i<=n; i=i\*2)  
    for (j=1; j<=i; ++j)

The statements in the above nested loop approximately run

**2n-1** times.

The variable j depends upon the value of i

### Rule 6

If the loop variables are independent then the total times a statement inside a nested loop is executed is equal to the product of the times the individual loops run

e.g. for (i=0; i<n; ++i)  
        for (j=0; j<m; ++j)

A statement inside the above nested loop will run **n\*m** times

### Other Guidelines

#### While loop related information

Complexity of “while” loop depend upon the initial entrance condition if it remains true for “n” iterations it will be “n+1”; Note here “1” will be added for the last time check of the condition. Here this will be clear to you if the some logical conditions are checked other then counters then all complexity will be based on scenario of the problem and nature of the logical condition.

### **Function Growth rate concept**

If some function  $f_1(x) > f_2(x)$  for positive values of  $x$  then the function  $f_1(x)$  is said to have greater growth rate than  $f_2(x)$ . For example  $f_1(x) = x^5$  and  $f_2(x) = x^6$  it is obvious that  $f_2(x)$  has greater growth rate ( $2^6 > 2^5$ ). This concept relates to complexity of algorithm, an algorithm having greater growth rate function means the algorithm has greater complexity here  $f_2(x)$  is more complex than  $f_1(x)$ .

### **Estimated Time 2.5 hour**

For all parts of the question to understand maximum time is 1.25 hours and for solution maximum time is 1.25 hour. It all depends upon your sheer concentration.

### **Question (4\*5)**

**Write piece of pseudo codes having the following time complexities:**

a)  $T_1(n) = n + \sum_{i=0}^n i$

b)  $T_2(n) = 6n (\log_{12} n) + 1 + (m/7) \log(m)$

c)  $O(z+m)$

d)  $T_3(n) = 5n \sum_{i=1}^n (1/i)$

**Note:** Where the base of “log” has not been mentioned take it as base “2”.