Fundamentals of Algorithms
CS502-Fall2011
SOLUTION ASSIGNMENT #3

Deadline

Your assignment must be uploaded/submitted at or before **2nd Jan 2012**

Uploading instructions

Please view the **assignment submission process** document provided to you by the Virtual University to upload the assignment.

Rules for Marking

It should be clear that your assignment will not get any credit if:

- The assignment is submitted after due date.
- The submitted assignment does not compile or run.
- **The assignment is copied.**

Objectives

This assignment will help you to understand the concepts of Knapsack Problem and edit distance problem in the paradigm of dynamic programming.

Guidelines

1. In order to attempt this assignment you should have full command on Lecture # 19 to Lecture # 26
2. In order to solve this assignment you have strong concepts about following topics
   - ✓ Edit distance Problem
   - ✓ Knapsack Problem

Recommended book for solving assignment

Cormen, Leiserson, Rivest, and Stein (CLRS) 2001, **Introduction to Algorithms**, (2nd ed.) McGraw Hill.

Estimated Time 4 hours

To understand the theme of both questions 90 minutes.Question1 solution implementation maximum time is 90 minutes and for Question2 solution implementation maximum time is one hour. It all depends upon your sheer concentration and devotion towards your lecture listening.

Question# (1)2 **(10)**

Use the following dynamic programming based recurrence edit distance to find the possible edit scripts while converting PHYSICIAN to STATISICIAN

$$E(i,j) = \min \begin{pmatrix} E(i-1,j)+1 & \\ E(i,j-1)+1 & \\ E(i-1,j-1)+1 & \text{if } A[i] \neq B[j] \\ E(i-1,j-1) & \text{if } A[i] = B[j] \end{pmatrix}$$

Solution

Some Basic assumptions:

E(i-1, j) + 1 repsents deletion and in table below "↓" has been used .

E(i, j-1) + 1 repsents insertion and in table "→"*has been* used .

E(i-1, j-1) + 1 if A[i]≠B[j] is for substitution and in table"↘" has been used.

E(i-1, j-1) if A[i]=B[j] is to maintain; means
previous adjacent diagnol value is transfered
with out any cost and here in table"↘"
has been used . Note here this symbol is being used for two
purposes one for substitution adding "1" in last cost and other for
"Maintain " without adding one .

Further these "• •" dots are path generating
hints think from there to backward for distinct
paths/edit-scripts.

↘ ↓
→ these are special cases just to think
you are getting minimum from all directions.

→ ⊗*These* are confusing points look
like as if there may be path but there is
breakage at this point.

Here more observations you can make you only need insertions , substitutions or maintain operations for the letters there will be no delete option for our given scenario; obviously we are talking of whole words not there substrings.

Solution table

		S	T	A	T	I	S	I	C	I	A	N
	0	→1● ●	→2	→3	→4	→5	→6	→7	→8	→9	→10	→11
P	↓1	↘1	↘● →●2	↘● →3	↘ →4	↘ →5	↘ →6	↘ →7	↘ →8	↘ →9	↘ →10	↘ →11
H	↓2	↘↓2	↘2	↘● →●3	↘● →●4	↘ →5	↘ →6	↘ →7	↘ →8	↘ →9	↘ →10	↘ →11
Y	↓3	↘↓3	↘↓3	↘3	↘ →4	↘● →●5	↘ →6	↘ →7	↘ →8	↘ →9	↘ →10	↘ →11
S	↓4	↘3	↘ ↓ →4	↘↓4	↘4	↘ →5	↘5	→6	→7	→8	→9	→10
I	↓5	↓4	↘4	↘↓5	↘↓5	↘4	→5⊗	↘5	→6	→7	→8	→9
C	↓6	↓5	↘↓5	↘5	↘ ↓ →6	↓5	↘5	↘ ↓ →6⊗	↘5	→6	→7	→8
I	↓7	↓6	↘↓6	↘↓6	↘6	↘↓6	↘↓6	↘5	↓ →6⊗	↘5	→6	→7
A	↓8	↓7	↘↓7	↘6	↘↓7	↘↓7	↘↓7	↓6	↘6	↓6	↘5	→6
N	↓9	↓8	↘↓8	↓7	↘7	↘ ↓ →8	↘↓8	↓7	↘↓7	↘↓7	↓6	↘5

Possible Edit Transcripts

FIRST

1+	1+	1+	1+	1+	0	0	0	0	0	0	=5
I	**I**	**S**	**S**	**S**	**M**	**M**	**M**	**M**	**M**	**M**	
_	**_**	**P**	**H**	**Y**	**S**	**I**	**C**	**I**	**A**	**N**	
S	**T**	**A**	**T**	**I**	**S**	**I**	**C**	**I**	**A**	**N**	

SECOND

1+	1+	1+	1+	1+	0	0	0	0	0	0	=5
S	I	I	S	S	M	M	M	M	M	M	
P	_	_	H	Y	S	I	C	I	A	N	
S	T	A	T	I	S	I	C	I	A	N	

THIRD

1+	1+	1+	1+	1+	0	0	0	0	0	0	=5
S	S	I	I	S	M	M	M	M	M	M	
P	H	_	_	Y	S	I	C	I	A	N	
S	T	A	T	I	S	I	C	I	A	N	

FOURTH

1+	1+	1+	1+	1+	0	0	0	0	0	0	=5
S	S	S	I	I	M	M	M	M	M	M	
P	H	Y	_	_	S	I	C	I	A	N	
S	T	A	T	I	S	I	C	I	A	N	

FIFTH

1+	1+	1+	1+	1+	0	0	0	0	0	0	=5
S	I	S	I	S	M	M	M	M	M	M	
P	_	H	_	Y	S	I	C	I	A	N	
S	T	A	T	I	S	I	C	I	A	N	

SIXTH

1+	1+	1+	1+	1+	0	0	0	0	0	0	=5
S	S	I	S	I	M	M	M	M	M	M	
P	H	_	Y	_	S	I	C	I	A	N	
S	T	A	T	I	S	I	C	I	A	N	

SEVENTH

1+	1+	1+	1+	1+	0	0	0	0	0	0	=5
I	S	I	S	S	M	M	M	M	M	M	
_	P	_	H	Y	S	I	C	I	A	N	
S	T	A	T	I	S	I	C	I	A	N	

EIGTH

1+	1+	1+	1+	1+	0	0	0	0	0	0	=5
S	I	S	S	I	M	M	M	M	M	M	

P	**_**	**H**	**Y**	**_**	**S**	**I**	**C**	**I**	**A**	**N**
S	**T**	**A**	**T**	**I**	**S**	**I**	**C**	**I**	**A**	**N**

NINETH

1+	**1+**	**1+**	**1+**	**1+**	**0**	**0**	**0**	**0**	**0**	**0**	**=5**
I	**S**	**S**	**I**	**S**	**M**	**M**	**M**	**M**	**M**	**M**	
_	**P**	**H**	**_**	**Y**	**S**	**I**	**C**	**I**	**A**	**N**	
S	**T**	**A**	**T**	**I**	**S**	**I**	**C**	**I**	**A**	**N**	

TENTH

1+	**1+**	**1+**	**1+**	**1+**	**0**	**0**	**0**	**0**	**0**	**0**	**=5**
I	**S**	**I**	**S**	**S**	**M**	**M**	**M**	**M**	**M**	**M**	
_	**P**	**_**	**H**	**Y**	**S**	**I**	**C**	**I**	**A**	**N**	
S	**T**	**A**	**T**	**I**	**S**	**I**	**C**	**I**	**A**	**N**	

ELEVENTH

1+	**1+**	**1+**	**1+**	**1+**	**0**	**0**	**0**	**0**	**0**	**0**	**=5**
I	**S**	**S**	**S**	**I**	**M**	**M**	**M**	**M**	**M**	**M**	
_	**P**	**H**	**Y**	**_**	**S**	**I**	**C**	**I**	**A**	**N**	
S	**T**	**A**	**T**	**I**	**S**	**I**	**C**	**I**	**A**	**N**	

Conclusion: The best possible way can be done at the cost of “5” for the conversion of “PHYSICIAN to STATISICIAN” by the dynamic approach result.

Question# 2 **(10)**

You are the project manager in multi-national company and you are asked to deploy the project for the firm to get the optimal solution and maximum profit in the given constraints

There are six projects i.e. Tele -Communication Project (P1), Hardware Devices Production Plant (P2), Software services for the educational institutes (P3), Distance learning Software Development Centre (P4) , Virtual cell phone development centre (P5) and Medical diagnoses devices enhancement centre (P6) .Your organization has Rs.150 billions to invest .If the cost of P1 is Rs.50 billion and profit earned is Rs.38 Billion, the cost of P2 is Rs.35 billion and profit earned is Rs.22 Billion, the cost of P3 is Rs.40 billion and profit earned is Rs.25 Billion ,the cost of P4 is Rs.65 billion and profit earned is Rs.58 Billion ,cost of P5 is Rs.55 billion and profit earned is Rs.45 Billion and the cost of P6 is Rs.60 billion and profit earned is Rs.47 Billion Your role is to maximize the profit in given amount to launch the projects. Determine the projects to be selected to earn the maximum profit using 0-1 Knapsack Show complete process.

Solution2

Some basic thinking points to solve smoothly:

First of all here one assumption is taken for clarity purpose:

In recurrence "w_i" is current weight under consideration and second "w"in recurrence is replaced as "J" just to make the idea clear and to make difference between two. More here J/w represents the capacity of knapsack. At end J=W means maximum capacity which we have here it is "150".

First think about the recurrence which is maximizing our profit at each cell as we are making optimized decision to fill the cell**. First case** in recurrence is trivial as nothing to select for weight "0" and capacity "0". **Second case** is again not difficult; if weight under consideration "w_i" is greater than capacity "J "you have to pick the very last row value in the same column calculated already .**Last case** of recurrence is selecting the maximum value from two .In which first value is in same column very last row and second value is calculated by adding the current value to the value which is actually at very last row having column index =J-w_i **which is the core point here to understand and "maximum value of two" will be selected for the cell.**

Simple hint : You are maximizing at each cell which ever you have up to Jth Limit/capacity of your knapsack .We have maximum J=W=150 so at end we are interested in to find the maximum profit for J=W=150 capacity that is our goal .

Note: Here capacity is Rs.150 billions . And each project cost is actually weight of item and profit earned is actually value of item. And detail is given in following table.

We will use the pneumonic specified instead of complete names of the projects.

Here discrete limits are taken for calculation wise efficiency however if we change limit spans selected values may change but over all maximum profit will remain same in all cases.

Project Name	Weight Limit J ->	0	15	30	45	60	75	90	105	120	135	150
P1	w_1 =50, v_1 =38	0	0	0	0	38	38	38	38	38	38	38
P2	w_2 =35, v_2 =22											
P3	w_3 =40, v_3 =25											
P4	w_4 =65, v_4 =58											
P5	w_5 =55,v_5 =45											
P6	w_6 =60,v_6 =47											

Project Name	Weight Limit J ->	0	15	30	45	60	75	90	105	120	135	150
P1	w_1 =50, v_1 =38	0	0	0	0	38	38	38	38	38	38	38
P2	w_2 =35, v_2 =22	0	0	22	22	38	38	60	60	60	60	60
P3	w_3 =40, v_3 =25											
P4	w_4 =65, v_4 =58											
P5	w_5 =55,v_5 =45											
P5	w_5 =55,v_5 =45											
P6	w_6 =60,v_6 =47											

Project Name	Weight Limit J ->	0	15	30	45	60	75	90	105	120	135	150
P1	w_1 =50, v_1 =38	0	0	0	0	38	38	38	38	38	38	38
P2	w_2 =35, v_2 =22	0	0	22	22	38	38	60	60	60	60	60
P3	w_3 =40, v_3 =25	0	0	22	25	38	47	63	63	63	85	85
P4	w_4 =65, v_4 =58											
P5	w_5 =55,v_5 =45											
P6	w_6 =60,v_6 =47											

Project Name	Weight Limit J ->	0	15	30	45	60	75	90	105	120	135	150
P1	w_1 =50, v_1 =38	0	0	0	0	38	38	38	38	38	38	38
P2	w_2 =35, v_2 =22	0	0	22	22	38	38	60	60	60	60	60
P3	w_3 =40, v_3 =25	0	0	22	25	38	47	63	63	63	85	85
P4	w_4 =65, v_4 =58	0	0	22	25	38	58	63	83	96	96	118
P5	w_5 =55,v_5 =45											
P6	w_6 =60,v_6 =47											

Project Name	Weight Limit J ->	0	15	30	45	60	75	90	105	120	135	150
P1	w_1 =50, v_1 =38	0	0	0	0	38	38	38	38	38	38	38
P2	w_2 =35, v_2 =22	0	0	22	22	38	38	60	60	60	60	60
P3	w_3 =40, v_3 =25	0	0	22	25	38	47	63	63	63	85	85
P4	w_4 =65, v_4 =58	0	0	22	25	38	58	63	83	96	96	118
P5	w_5 =55,v_5 =45	0	0	22	25	45	58	67	83	103	103	118
P6	w_6 =60,v_6 =47											

Project Name	Weight Limit J ->	0	15	30	45	60	75	90	105	120	135	150
P1	w_1 =50, v_1 =38	0	0	0	0	38	38	38	38	38	38	38
P2	w_2 =35, v_2 =22	0	0	22	22	38	38	60	60	60	60	60
P3	w_3 =40, v_3 =25	0	0	22	25	38	47	63	63	63	85	85
P4	w_4 =65, v_4 =58	0	0	22	25	38	58	63	83	96	96	118
P5	w_5 =55,v_5 =45	0	0	22	25	45	58	67	83	103	103	118
P6	w_6 =60,v_6 =47	0	0	22	25	47	58	67	83	103	105	118

Selected Projects are P1 , P2 and P4 that will give the profit of Rs.118 billion.

Tele -Communication Project (P1) +Hardware Devices Production Plant (P2) + Distance learning Software Development Centre (P4)=Rs.118 Billion Profit